Nonlinear principal component analysis of the tidal dynamics in a shallow sea

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[1] A nonlinear, neural-network-based extension of the principal component analysis (PCA) is applied to the water level and current fields in a shallow tidal sea at the German North Sea coast. Contrary to the linear PCA, which tends to split patterns in the data among several modes difficult to interpret, the nonlinear PCA enables to identify the nonlinear spatial patterns in the data with only a single mode. The first nonlinear principal component (PC) corresponds well with the joint probability distribution of the linear PCs and can be argued to represent a 'typical' tidal cycle in the study area. **Citation:** Herman, A. (2007), Nonlinear principal component analysis of the tidal dynamics in a shallow sea, *Geophys. Res. Lett.*, *34*, L02608, doi:10.1029/2006GL027769.

1. Introduction

[2] Medium-term (∼decades) variability of water levels and currents is an important factor influencing many shallow water phenomena, including sediment transport processes and load on the coastal protection structures. Exhaustive analysis of this variability over large areas with complicated topography is possible so far only with high-resolution numerical models. However, the amount of data produced by those models is often prohibitively large. In such cases the further application and interpretation of the modelling results must be combined with an efficient data reduction technique. One of the most popular geostatistical data reduction tools is the principal component analysis (PCA), in which the analyzed data set is decomposed into a number of time-invariant spatial modes and corresponding principal components (PCs), describing the time evolution of the amplitude of the modes. The strength of PCA lies in the significant reduction of the dimensionality of the data without loss of important information. However, applied to systems with strongly nonlinear behaviour PCA has a number of obvious drawbacks, the most important of which is that nonlinearities 'hidden' in the data are scattered among several PCs, making the proper interpretation of the results and the identifying of the physical patterns impossible and unnecessarily increasing the number of PCs required to reproduce the desired amount of variance of the original data set. Another drawback is the behaviour of PCA when applied to data sets containing propagating features, which get similarly scattered among several PCs. Although the last shortcoming can be partially accounted for by means of other linear techniques, like e.g. the complex PCA or the multichannel singular spectrum analysis, a more natural solution to the above mentioned problems is a nonlinear extension of PCA (NLPCA) proposed originally by *Kramer* [1991], the main idea of which is to relax the requirement that the sought for 'axes of concentration' of the data must be straight lines and to allow them to form curves (or even, in the case of circular NLPCA, closed curves) that adjust their shape to optimally pass through the data clusters. NLPCA has an important advantage over the linear techniques mentioned above, which produce linearly independent, but statistically correlated modes. For the theory of NLPCA and its applications in climate studies, the reader is referred to *Hsieh and Tang* [1998], *Monahan* [2000], *Monahan et al.* [2001], *Rattan and Hsieh* [2004], *Hsieh* [2001, 2004b], and *Rattan et al.* [2005]. Suitability of NLPCA to studies concerning the medium-term variability of hydrodynamic processes in the coastal zone is demonstrated below.

[3] The motivation for the NLPC analysis presented in this study originated in the analysis of the PCA results of water levels and currents in a shallow, tidal sea (Figure 1). Although the linear PCs are by definition linearly independent and uncorrelated, the joint probability distribution of the PC-pairs reveals the existence of certain statistical relationships between them (Figure 2). This suggests the possibility of describing the analyzed data sets in another, more appropriate than the linear PCA way.

2. Data and Methods

[4] The study area is located at the German coast and consists of the catchment areas of the tidal inlets between three of the East Frisian Islands, separating the Wadden Sea from the North Sea (Figure 1). The high-resolution hydrodynamic modelling in this area, with realistic forcing, has been performed within a large research project by means of the 2D version of the Delft3D model [*Roelvink and van Banning*, 1994], as described in detail by A. Herman et al. (A new approach towards modelling of a medium-term dynamics in a shallow tidal sea, based on combined physical and neural network methods, submitted to *Ocean Modelling*, 2006, hereinafter referred to as Herman et al., submitted manuscript, 2006). For the purpose of the present analysis the simulated water level and current fields from the years 1962–1963, 1985, and 2002 (detailed statistical analysis of the input data used for modelling showed that the probability distribution of the analyzed parameters over these years is representative for the variability of those parameters in the four decades studied) have been saved hourly in N_p = 3454 data points (every 5th point of the

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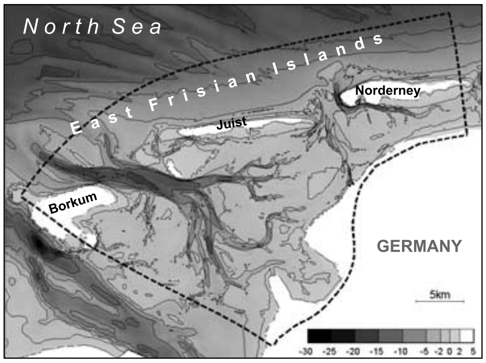

Figure 1. Bottom topography (in m) of the analyzed part of the German Wadden Sea. Dashed line shows the boundary of the study area.

computational grid; only points 'wet' at least 10% of the time have been taken into account), resulting in $N_p \times N_t$-matrices (N_t = 35043), real-valued for water levels ($\mathbf{W} = [W_{pt}]$) and complex-valued for currents ($\mathbf{V} = [U_{pt}] + i[V_{pt}]$, where i is the imaginary unit and U, V are the cartesian velocity components). All exception values, corresponding to drying of the tidal flats, have been replaced with the bottom level in the **W** matrix and with zeros in the **V** matrix. The distribution of variance of the original data sets among the five leading linear modes resulting from the PCA is shown in Table 1. A prevailing part of the variance of both water levels and currents is represented by the first mode. In both cases the three leading modes and corresponding PCs are used in the NLPC analysis presented further—a choice based on the visual examination of the modes and of the spatial distribution of the percentage of variance explained by them. The further modes are very irregular spatially, have amplitude much lower than the first ones, and are 'important' only locally, mainly in points staying dry over most of the tidal cycle; see also discussion of Herman et al. (submitted manuscript, 2006). Taking into account the further modes complicates the whole picture without bringing to light valuable information.

[5] The chosen PCs of water levels and currents are used as input for the circular NLPCA, performed with the NLPCA.cir MATLAB package developed by *Hsieh* [2004a]. For each of the two analyzed parameters an ensemble of 20 five-layer autoassociative neural networks (NNs) has been set up, with 3 hidden neurons in the encoding and decoding layers and a weight penalty parameter of 1.0 to avoid overfitting. In the case of currents the real and imaginary parts of the PCs were treated separately. Thus, the NNs for water levels and currents have three and six input/output neurons, respectively. To enforce a closed-curve solution, the constrained form of the cost function has been used. The member networks of each ensemble have been trained with various initial parameters to prove the sensitivity of the networks to the values of these parameters and to ensure the repeatability of the results (see *Monahan and Fyfe* [2007] for the discussion of the robustness and uniqueness of the NLPCA results). In both cases over 90% of the ensemble members gave very similar results, with the cost function J varying within 10%. From each of the two ensembles the network resulting in the lowest J value has been chosen for the final analysis.

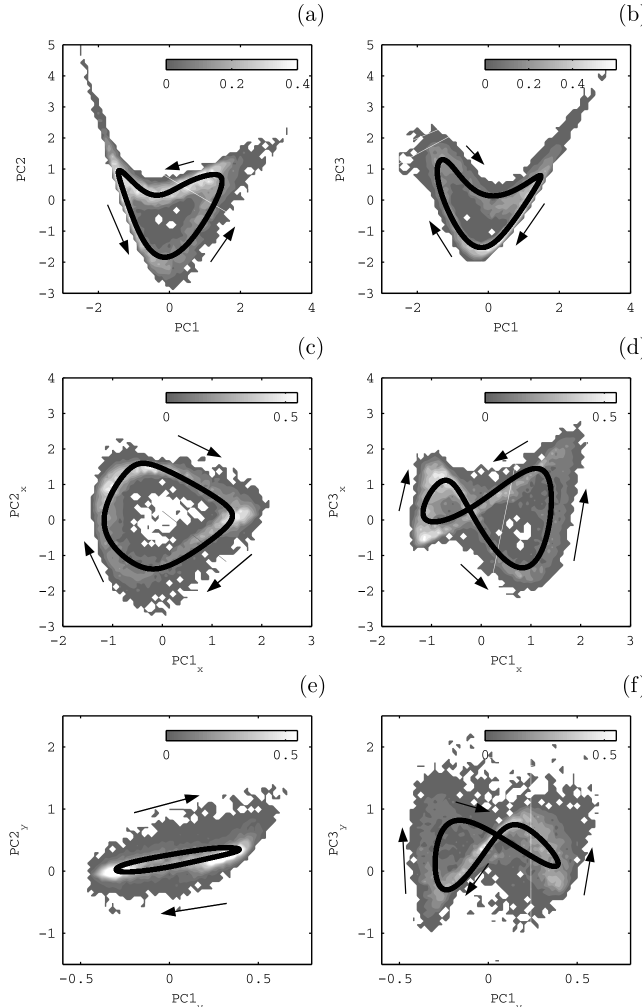

Figure 2. Discrete joint probability density (in %; class width 0.1) of the three leading PCs of (a–b) water levels and (c–f) currents (real parts, Figures 2c and 2d; imaginary parts, Figures 2e and 2f), normalized to unit standard deviation. Overlayed is the first NLPC (black lines). Arrows show the direction in which the curves are 'drawn' during a tidal cycle.

3. NLPCA Results

[6] The first nonlinear PCs (NLPCs) of water levels and currents explain 97.4% and 89.5% of the variance of the respective data sets. Overlayed onto the diagrams showing

Table 1. Percentage of Variance Explained by the First Five Linear Modes of Water Levels and Currents

	Water Levels		Currents	
Mode	Separate	Cumulative	Separate	Cumulative
1	97.177	97.177	84.471	84.471
2	1.803	98.980	7.263	91.734
3	0.638	99.618	1.966	93.699
4	0.144	99.762	1.508	95.208
5	0.052	99.814	0.677	95.885

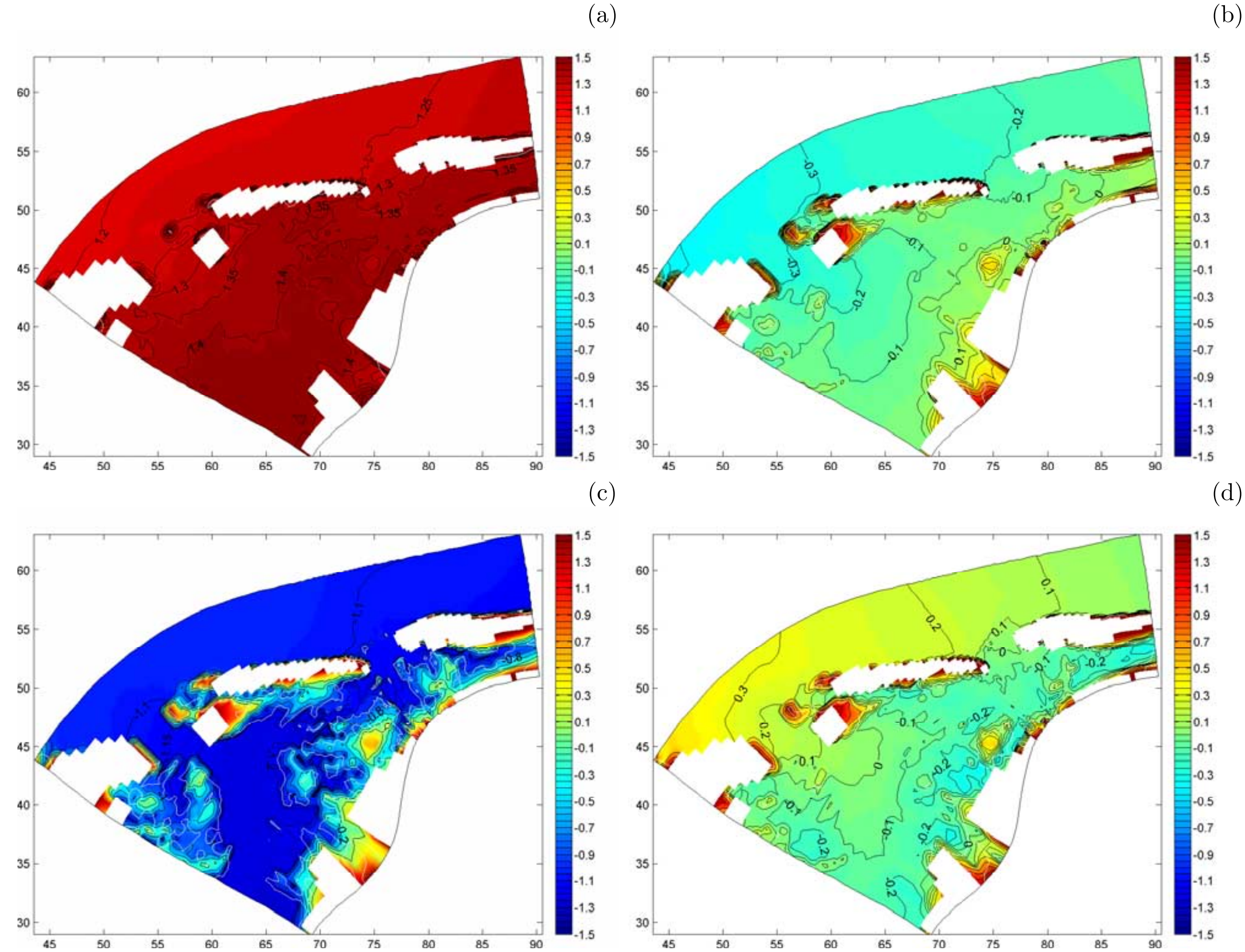

Figure 3. The first nonlinear mode of water levels at the four extremes of the first NLPC of water levels (Figure 2a): (a) high water, (b) ebb, (c) low water, and (d) flood. Average values, that had been extracted from the data set prior to the PCA, have been added to the solution to make it represent a 'typical' course of water levels during one tidal cycle. Axes labelling in km. White contours mark the areas lying above water level.

the discrete joint probability distribution of the linear PC pairs (Figure 2), the first NLPCs pass directly through the regions of the highest data concentration. In both cases the first NLPC forms a complicated loop in the space spanned by the linear PCs (arrows in the diagrams in Figure 2 show the direction in which the curves are 'plotted' during a tidal cycle). The shape of these loops and their asymmetry are a clear indication that the paths drawn in the phase space during ebb and flood are very different. A hypothetical system reacting in the same way on falling and rising water level would result in NLPCs with overlapping fragments corresponding to ebb and flood, which is not the case in the data set analyzed.

[7] Contrary to the results of the linear PCA, NLPCA gives time-varying modes. The curves drawn by the first NLPC of water levels and currents can be argued to represent the course of the 'typical' or 'most probable' tidal cycle, with each point on these curves representing the state of the system—the shape of the water surface and the current field, respectively—at a given tidal phase. As an example, four of those states, corresponding to the four sharp bends of the first NLPC curve of water levels in Figure 2a, are shown in Figure 3. A proper interpretation of these plots is possible if one considers that the first linear PC of water levels depends strongly (with a correlation coefficient of 99.7%) and almost linearly on the mean water level in the study area (Herman et al., submitted manuscript, 2006). Thus, with help of Figure 2, it is easy to relate the contour plots in Figure 3 to high water, ebb, low water and flood. (Figures 3b and 3d do not correspond to the times of strongest tidal currents and maximal water level gradients, but rather to the tidal phases at which the system shifts direction in the phase space.)

[8] Contrary to the linear modes, which do not resemble any physically meaningful patterns observable in a real world, the time series of the first nonlinear modes can be shown to be both qualitatively and quantitatively in agreement with the observed variability of water levels and currents in the study area. An animation of the intermediate states linking the four extremes presented in Figure 3 clearly illustrates this fact (Animation S1 in the auxiliary material).[1]

[1]Auxiliary materials are available in the HTML. doi:10.1029/2006GL027769.

The most striking difference between the linear and nonlinear modes concerns the ability to reproduce the propagation of the tidal wave along the coast and into/out of the tidal inlets. Whereas it can be directly observed in the NLPC animation, it cannot be recognized in any of the single PC modes, in particular not in the first water level mode, although it represents over 97% of the total variance. Another one of the most characteristic features of the tidal regime of the study area—accurately predicted by the first NLPC—is a relatively steep slope of water surface at high water: 15–20 cm from the mainland coast to the open sea (Figure 3a). Apart from the geometry of the tidal inlets, the prevailing winds have presumably a noticeable influence on this effect. Also, the first NLPCs correctly predict higher alongshore (to the north of the islands) water level gradients and stronger currents by ebb than by flood. The current asymmetries in the tidal channels are (at least qualitatively) correctly reproduced by the first NLPC as well.

4. Discussion and Conclusions

[9] The NLPCA has proved to be a useful data analysis tool in climatology and atmospheric sciences. This study examines the aspects of NLPCA applied in coastal hydrodynamics—to the best of the author's knowledge it is the first application of this technique to the analysis of the spatial and temporal variability of water levels and currents in the coastal zone (*Ruessink et al.* [2004] applied this technique to study bathymetry changes and propagation of sand bars nearshore). The results presented above clearly indicate the suitability of this tool to problems concerning water circulation, at least—like in the discussed case—in relatively small areas subject to strong external forcing, where internal, mainly geometrical constraints exist that leave only little space for possible 'ways of response' to that forcing. These constraints limit the possible modes of variability in the system, but at the same time lead to its strongly nonlinear behaviour, as many studies concerning the tidal inlet dynamics demonstrate. In the analyzed case the NLPCA has obvious advantages over its traditionally used linear counterpart.

[10] From the discussion in the previous section it follows that the NLPCA results for water levels can be interpreted in a way analogous to the mean tidal curve traditionally used in many practical applications. The crucial difference, revealing the power of NLPCA as compared with other methods, is that instead of a time history of water level in a single point, spatial and temporal water level variability in the whole study area is described in a compact way, indirectly taking into account the whole spectrum of processes influencing this variability, e.g. prevailing winds. Of course, although the analogy between the mean tidal curve and the NLPC may be useful, their very different basics must be kept in mind. Whereas the first one is an arithmetic average of a number of tidal cycles in a given point, the second one should be rather interpreted in a mean-root-square-error sense, accordingly to the definition of the cost function used in the neural networks as a measure of their fitness.

[11] In summary, the analyzed case demonstrates that the NLPCA can be regarded not only as an interesting way of looking at the representative hydrodynamic conditions in the coastal zone, but also as a useful tool, at least complementary to the linear PCA, helpful by identifying physical patterns in the data.

[12] **Acknowledgments.** The work presented here has been part of the project MOSES ("Modelling of the medium-term wave climatology at the German North Sea coast"; project 03 KIS 040) financed by the German Federal Ministry for Education and Research (BMBF) under the umbrella of the German Coastal Engineering Research Council (KFKI).

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