HARBASINS

HARBASINS Report:

Set-up of a hydrodynamic model for the Ems-Dollard estuary

Prepared by: Gerald Herrling and Hanz D. Niemeyer
Lower Saxony Water Management,
Coastal Defence and Nature Conservation Agency
-Coastal Research Station-

HARBASINS is a project funded under the European Regional Development Fund INTERREG IIIB North Sea Region Program – A European Community Initiative concerning Trans National Co-operation on Spatial Development 2000-2006.

Document history

Revisions

Version	Status	Date	Name	Changes
1.0	draft	10.08.07	Herrling, Niemeyer	
2.0	final	19.03.08	Herrling, Niemeyer	Revision, plots improved

Distribution

This document has been sent to:

Version	Date sent	Name	Position
2.0	13.06.2008	Frank Zijp	Project leader
2.0	13.06.2008	HARBASINS webpage	

Authors:
Gerald Herrling
Hanz Dieter Niemeyer

Table of Content

List of Figures

1 Introduction

In coastal areas and particularly in estuaries or areas such as the Wadden Sea, there is a lack of procedures for the identification of ‘Heavily Modified Water Bodies’ (HMWB) according to the water framework directive (WFD) of the European Community. Aim of the investigation is to identify water bodies by comparable standardized methods, e.g. by applying mathematical models.

Currently, the assessment criteria concentrates on the area of impact, but this approach may be insufficient when alterations to current regimes may affect salinity levels and sediment transport in areas outside of the direct impact zone.

Aim of work package 4 “Hydro-morphological Impacts and Pressures” in the HARBASINS project is to generate process-based knowledge on these effects by high-resolution mathematical modelling in combination with the analysis of hydro- and morphodynamical parameters. Ultimately, it is intended to establish a modelling strategy to identify the spatial scale of potential HMWBs.

The Ems-Dollard estuary covering the area from the East Frisian Islands as far upstream as the tidal barrier at Herbrum in the Lower Ems is selected as the study area for this purpose.

This report concentrates on the set-up process of a 2DH hydrodynamic model as regards the implementation of the computational grid, the bathymetry, the open boundary conditions and the setting of different input parameters. The model is calibrated by fine-tuning numerical and physical parameters, e.g. the bottom roughness. The quality of the model is verified by comparing computed and measured water levels at gauge locations along the estuary.

The configuration of the hydrodynamic model serves as a basis for follow-up calculations of sediment transport and bottom evolution as well as for investigations applying historical topographic reconstructions that will be reported in subsequent reports.

2 Area of investigation

The investigation area is located at the Dutch-German North Sea coast and covers the Ems-Dollard estuary as a whole. The seaward limit is close to the 20 meter depth-line in the outer estuary; the landward limit is at the tidal barrier at Herbrum in the Lower Ems. The study area is marked by all geomorphological features characteristic for this type of coastline: deep tidal channels and inlets, inter-tidal flats and the inner estuarine environment (Fig.1).

The width of the Ems increases in flow direction from about 60 m at Herbrum to about 600 m at Pogum. Downstream of Pogum, the Dollard Bay that originated due to the impacts of medieval storm surges is part of the estuary. At Knock, about 70 km downstream of the tidal barrier, the Ems flows in the outer estuary; the width of the estuary is here about 3500 m.

At Eemshaven, the outer estuary splits in the Wester and the Easter Ems, but nowadays the Easter Ems is no longer of the estuary. The Easter Ems has separated from the Ems estuary as a tidal inlet with an own catchment area. The estuarine tidal volume exchanges through the tidal inlet in West of Borkum, while the inlet at East of Borkum only provides the water exchange of the eastern tidal flats (SCHUBERT, 1970).

The actual mean tidal range in the Ems estuary has a bandwidth between 2.4 m at the island of Borkum increasing to its maximum of 3.5 m at Papenburg and decreasing upstream to 2.7 m at the tidal border at Herbrum (Fig. 2).

The salinity remains nearly constant at Borkum for mean tidal and freshwater conditions. Further upstream it reduces gradually up to Leer. Contradictory, the concentration of suspended matter increases upstream of Borkum reaching its maximum of about 400 g/m³ between Jemgum and Leer (JONGE, 2000).

Water levels are recorded at several fixed tidal gauges along the estuary. Water level time series of the gauge stations of Borkum Fischerbalje, Knock, Emden, Pogum, Leerort, Papenburg and Herbrum were used for the calibration and validation of the mathematical model.

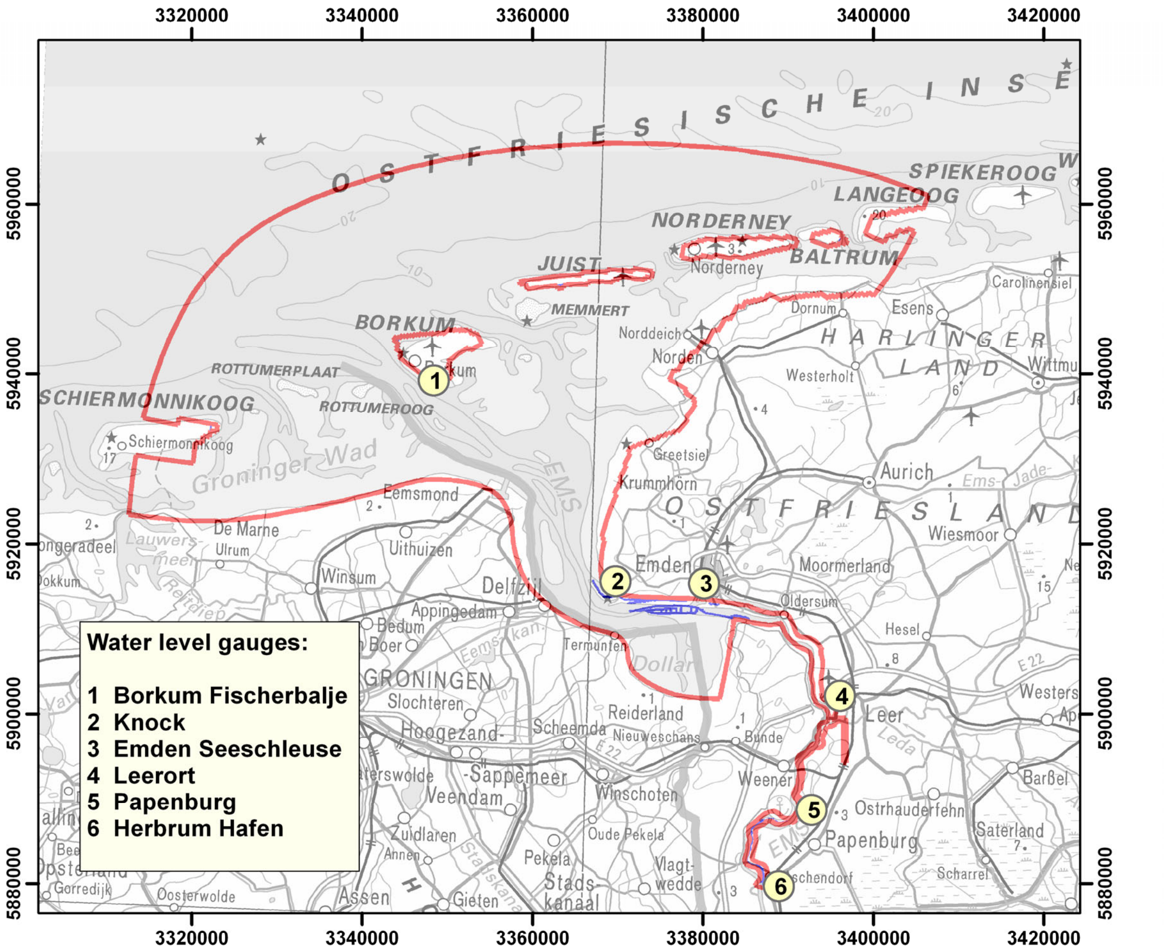

Fig. 1: Investigation area and water level gauges

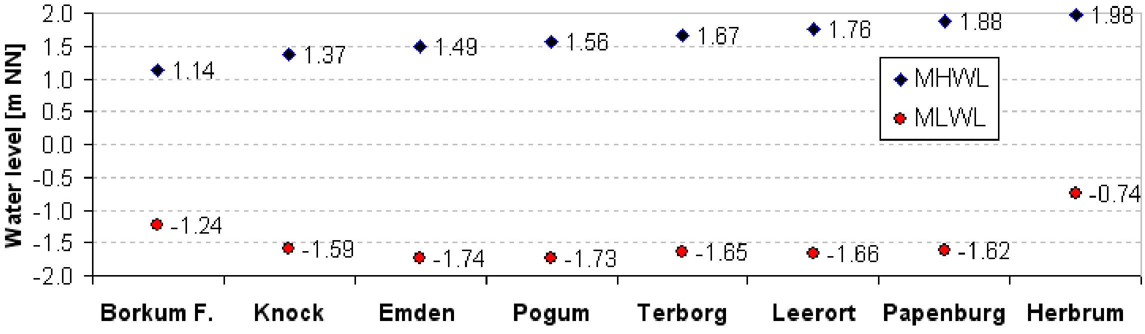

Fig. 2: Mean high water and mean low water levels (MHWL and MLWL) at gauges along the Ems-Dollard estuary for the period from 2001 to 2005

3 Set-up of the hydrodynamic model

3.1 General

3.1.1 Conventions and definitions

The coordinate system used for the model generation and any horizontal positioning in the report is the German Gauss Krueger coordinate system, zone 3° E. The x-axis is pointing East, the y-axis is pointing North. The water levels and the bottom depths are given in meters with reference to Normal Null (mNN), which is approximately Mean Sea Level (MSL). All time data in the report is referred to Greenwich Mean Time (GMT) based on conventions in the modeling system.

3.1.2 Modeling system

The deterministic-mathematical model is set-up with the modeling system Delft3D (DELFT HYDRAULICS, 2006). The applied hydrodynamic module Delft3D-Flow is able to simulate two- or three-dimensional unsteady flow and transport phenomena resulting from tidal or meteorological forcing, including the effect of density differences due to a non-uniform temperature and salinity distribution. The program is based on three dimensional shallow water equations, the continuity equations and the transport equations for conservative constituents. The set of partial differential equations in combination with appropriate initial and boundary conditions are solved with a finite difference scheme on an orthogonal-curvilinear grid.

For this study, the model was applied in the two-dimensional, horizontal mode, i.e. the modeled current velocities are depth-averaged and density driven flow in the vertical is neglected. Meteorological boundary conditions, i.e. wind forcing, are neglected, too.

3.2 Computational grid

The modelling approach intended to include the Ems-Dollard estuary as a whole. A computational grid was generated covering the western East Friesian Islands at the seaward limit up to the tidal barrier at Herbrum at the upper limit of the estuary. By constricting the grid lines towards the upper end of the estuary, the spatial resolution could be increased significantly along the inner estuary. For graphically representing the alignment of the grid lines, the resolution of the computational grid was fourfold decreased (Fig. 3a).

The spatial resolution of the computational grid is a representative dimension of an individual grid cell and is defined as the square root of the cell area. The resolution of the curvilinear grid is in the order of 800 m (1000 m x 700 m) at the seaward boundary and reaches up to 20 m (25 m x 15 m) in the river Ems (Fig. 3b). The generated computational grid comprises a total of 1898 x 228 = 432,744 computational points, with about 97,087 active points.

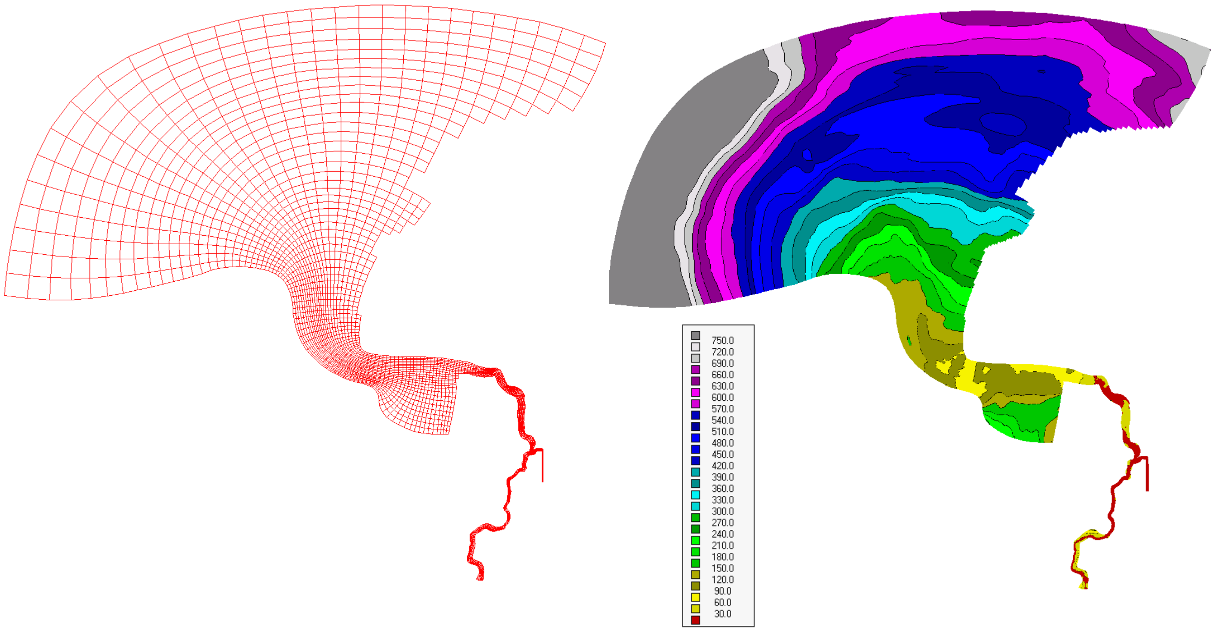

Fig. 3: a) Computational grid with fourfold decreased resolution for improved graphical representation; b) Grid resolution in meters of the applied computational grid

3.3 Model bathymetry

3.3.1 Available topographic data

Detailed bathymetrical information on the area of investigation is necessary to set-up the mathematical model. Data of recent topographic surveys has been required from the Federal Waterway Agency Emden and from the survey data base of the Coastal Research Station. The available data had been taken by different measuring methods such as common echosounding, side-scan-sonar and airborne laserscan. Data thus obtained was based on different reference and coordinate systems that had to be converted and processed using GIS software to further implement an adequate bathymetry in the modelling system.

Basic data have been gained by side-scan-sonar in 2005 and by sounding in 2004 for sub- and intertidal areas. Inter- and supratidal areas have been surveyed by airborne laser-scanning in 2005. Airborne laser-scanning produces data with an original resolution of 1 x 1 meters. The enormous amount of data had to be reduced by decreasing the data resolution to 5 x 5 meters in order to increase the effectiveness in data handling.

All bathymetric data was structured and interpolated onto the computational grid applying different interpolation methods depending on the spatial density of the bathymetric data in relation to the spatial resolution of the grid.

3.3.2 Schematization of the bathymetry

Bottom depths range from about -25 mNN at the tidal inlets and the seaward boundary of the model to a fictive elevation of +9 mNN landwards of the main dykes (Fig. 4). The Dollard Bay (Fig. 5a) and exemplarily a part of the Lower Ems from Jemgum to Gandersum (Fig. 5b) are enlarged with respect to an improved presentation.

The river Leda, a tributary stream of the Ems, is schematized up to its tidal limit by a rectangular channel. Reason for this schematization was the exceedance of the maximal number of computational points. The extension and the total volume of the channel are approximated by considering the representative tidal prism of about 3 millions cubic meters. The size and volume of the tributary system has been further adjusted in the calibration process.

3.3.3 Schematization of hydraulic structures

Hydraulic structures as groins, weirs, summer dykes or training walls are obstacles for currents generating transitions from flow contradiction to flow expansion and vice versa leading to energy losses and changing flow directions.

In most cases, the size of those hydraulic structures is small compared to the size of a computational grid cell. As a consequence the solid structure cannot be schematized in the model bathymetry. In order to model their impact on the flow, the flow through a computational cell is blocked or an energy loss term is added to the momentum equation. Detailed information about the numerical implementation is given by DELFT HYDRAULICS (2006).

Summer dykes in the Lower Ems are not overtopped by ordinary tides but only during storm surges. Since aim of the investigations is to model ordinary tidal conditions, these dams are parameterized in the model by so called “thin dams” which block the flow completely.

Groins along the Lower Ems are parameterized by “2D-gates” which do not block the flow but generate an energy loss. The energy loss is determined by the relation of height of the groin to the water depth and can additionally be adjusted by a coefficient. The elevations of the groins were defined according to available data.

The Geise training wall separating theEmden waterway from the tidal flats of the Dollard Bay is also parameterized by the above mentioned “2D-gates” (Fig. 5a). Large parts of the solid construction are flooded at Mean High Water.

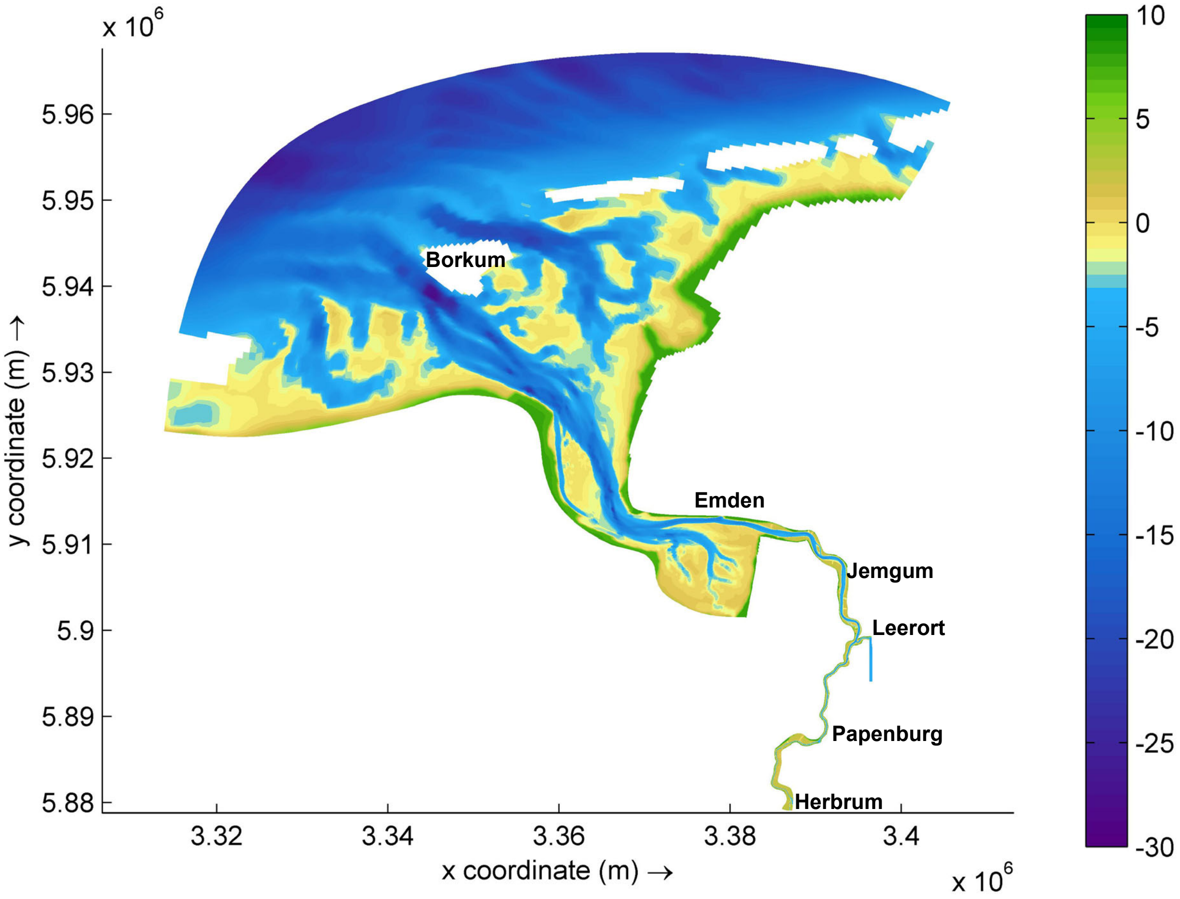

Fig. 4: Overview of the model bathymetry [mNN]

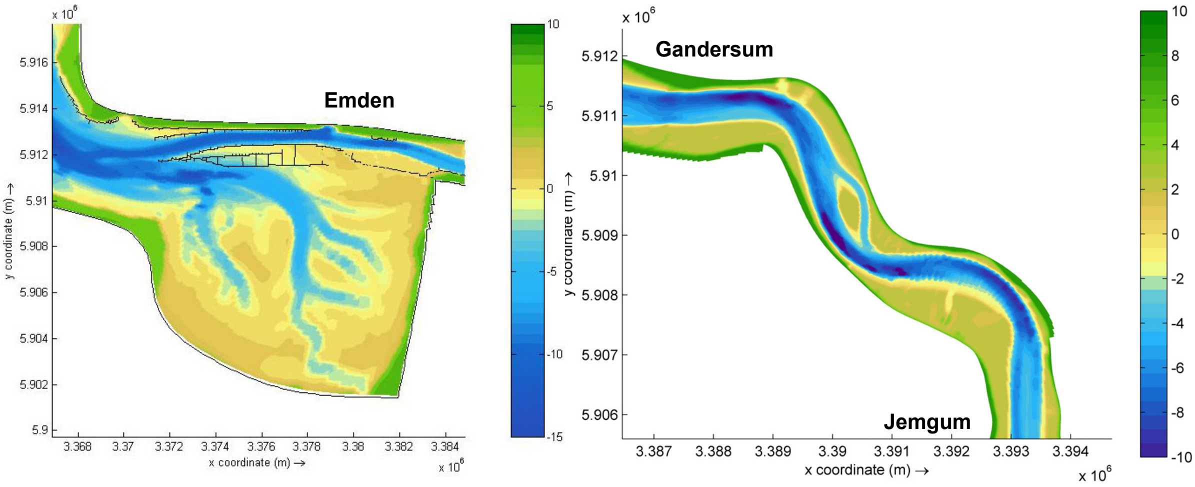

Fig. 5: Enlarged model bathymetry [mNN]: a) Dollard and b) Lower Ems between Jemgum and Gandersum (downstream)

3.4 Open boundary conditions

3.4.1 Sea boundary conditions

To obtain the tidal forcing at the open offshore boundary of the Ems-Dollard Model, the model is nested into the existing “German Bight Model” (WL|DELFT HYDRAULICS 1997). This overall model covers the coastal waters from Terschelling in the Netherlands to North of Esbjerg in Denmark. The computational grid of the German Bight Model is presented by a blue grid; the extension of the nested Ems-Dollard Model is shown in red colour (Fig. 6).

The water levels at the sea boundary (pink line) of the German Bight Model are prescribed by 29 tidal constituents that in turn are gained from a nesting procedure with the larger “North Sea Continental Shelf Model” (VERBOOM et al. 1992).

Tidal predictions of the German Bight Model allow the generation of open boundary conditions at the seaward limit (green line) of the Ems-Dollard Model. Seawards of the East Frisian Islands the boundary conditions are prescribed by time series of water levels. Two short boundary sections located near the tidal watersheds landwards of the islands of Schiermonnikoog and of Langeoog are determined by time series of tidal current velocities instead of water levels resulting in a more stable solution because these boundary locations fall periodically dry at lower tidal water levels.

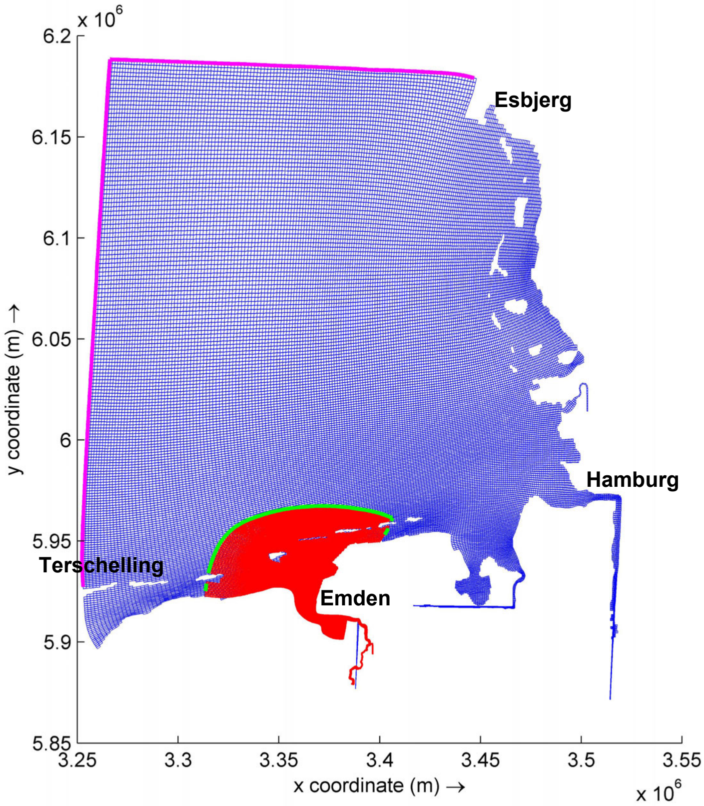

Fig. 6: Nesting of the Ems-Dollard Model (red) into the overall German Bight Model (blue)

The selected type of boundary conditions, i.e. the tidal forcing obtained from the overall German Bight Model, only represents the astronomical tide without any meteorological effects. Thus the observed tide, i.e. measured water levels, will to some extend differ from

the astronomical tide generated by the model due to effects like wind or variation in atmospheric pressure.
For this reason, the time period selected for tidal predictions in the calibration process (Chapter 4) is characterized by moderate meteorological conditions with low wind forces of less than Beaufort 4.
Simulations for stronger or even extreme meteorological conditions resulting in significant local set-up or set-down due to wind forcing would require a different approach for obtaining appropriate offshore boundary conditions.

3.4.2 Harmonic analysis

Harmonic analysis is a method to decompose a given tidal signal in its representing partial tides, i.e. astronomical constituents. For any coastal location, each partial tide has a particular amplitude and phase.

The comparison of astronomical constituents based on the analysis of computed time series of water levels on the one hand and observed ones on the other provide an important criterion about the quality of a model. To determine which constituents are over- or underestimated in the computed tidal signal, the amplitudes and phases of each computed partial tide are compared to the related amplitudes and phases of the observed partial tides.

The offshore boundary conditions of the Ems-Dollard Model are generated by the German Bight Model. In order to assess the reliability of the boundary conditions and hence the quality of the German Bight Model, the constituents based on the harmonic analysis of computed and measured water levels are compared for the location of the tidal gauge Borkum-Fischerbalje. Its location is close to the offshore boundary of the Ems-Dollard Model. The time series used for the analysis cover the period of one year, respectively 1987 and 2005. Astronomical constituents due to the observations of the year 1987 at Borkum Fischerbalje have been evaluated by PANSCH (1989). The observations of 2005 and the computed water levels of 1987 and 2005 are analyzed by means of a harmonic analysis applying the software toolbox MATLAB.

The comparison of the amplitudes of the 13 principal constituents at Borkum-Fischerbalje in between the years 1987 and 2005 shows good agreements, respectively for the measured and computed values (Fig. 7). There is no mayor variation between 1987 and 2005.

More obvious are the mismatches between the values based on modeled and observed tides. The harmonic analysis reveals that the amplitude of the main lunar tide M2 is overestimated by about 4.6 cm (4%) in case of the modeled values (1987 and 2005) in comparison to the observations of the same years. The larger lunar elliptic tide N2 is underestimated by about 1.5 cm (9%) by comparing computed values with observations.

The amplitudes of the secondary components MU2 and NU2 show larger percental discrepancies between measured and modeled values, respectively 1.9 cm (16%) and 5.3 cm (79%), but are less weighted on the composed tidal signal due to smaller amplitudes. For all other computed and observed partial tides the agreement of the amplitudes is quite good.

Comparing the tidal phases for each particular partial tide, the values rather spread (Fig. 8). For many of the 13 constituents the lags between the observed and computed tidal phases keep within a span of 60 degrees. For some particular constituents, mainly O1, M4, MS4 and 2MS6, there is poor quality achieved.

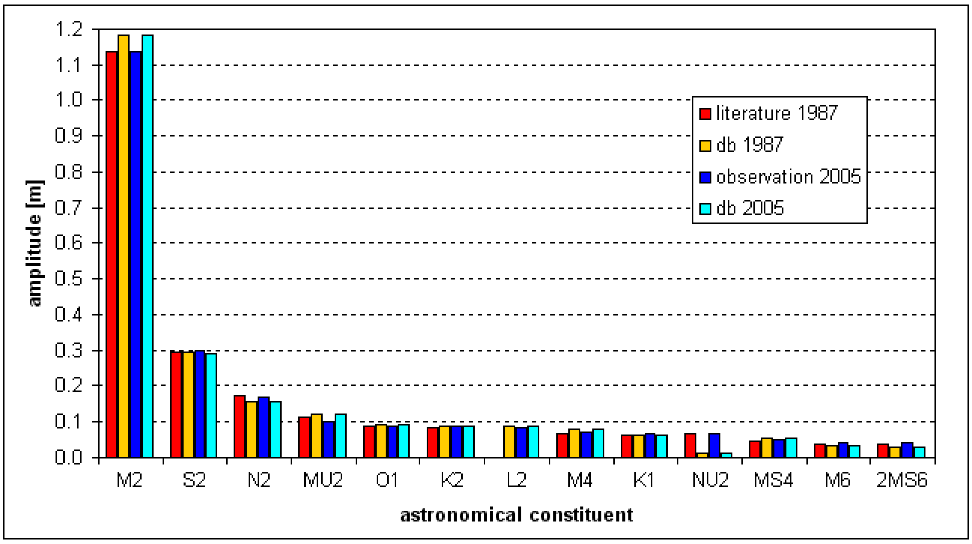

Fig. 7: Amplitudes of the 13 principal tidal constituents at Borkum based on yearly time series due to literature 1987 (PANSCH 1989), German Bight Model 1987, observations 2005 and the German Bight Model 2005

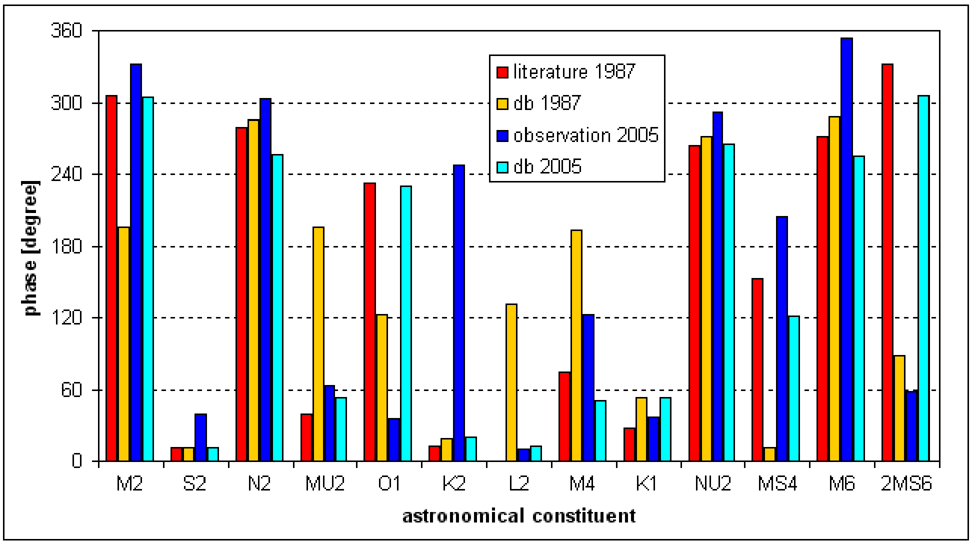

Fig. 8: Phases of the 13 principal tidal constituents at Borkum based on yearly time series due to literature 1987 (PANSCH 1989), German Bight Model 1987, observations 2005 and the German Bight Model 2005

The harmonic analysis reveals that some of the partial tides that are based on the tidal signal generated by the German Bight Model do not match the partial tides based on observations. The differences are partly caused by a relatively low resolution of the topographic schematization in the near coast area of the German Bight Model. An intermediate model with a numerical resolution that is in between the resolution of the German Bight and the Ems-Dollard Model would be very helpful generating more accurate offshore boundary conditions. But for the purpose of this investigation, the quality of the offshore boundary conditions being generated by the German Bight Model is assumed being sufficient.

3.4.3 River discharges

The discharges of the river Ems are gained from generally available data bases as daily means. The mean maxima, mean and mean minima discharges of the river Ems are respectively 375 m³/s, 81 m³/s and 16 m³/s with respect to long time series.

The river Leda, a tributary of the Ems, has a mean yearly discharge of 25 m³/s and discharges into the Ems at Leer. The smaller Westerwoldsche Aa discharges in the southern part of the Dollard Bay with a yearly average of 10 m³/s. Tributaries or drainage channels with minor discharges are not implemented in the model.

Daily time series are not available for the discharges of the rivers Leda and Westerwoldsche Aa. For this reason it is assumed that the relation of the daily discharge and the yearly mean discharge of the river Ems can be correlated to the corresponding relation of the Leda and the Westerwoldsche Aa.

For the calibration period from 22nd to 25th of June 2005 the discharge of the Ems is set according to the prevailing records; the discharges of Leda and Westerwoldsche Aa are related as mentioned above. For this period the discharges of the Ems, Leda and Westerwoldsche Aa are set respectively 32 m³/s, 10 m³/s and 4 m³/s being significantly lower than the yearly means.
For the validation period from 29th of June to 14th of July 2005 the discharges of the Ems, Leda and Westerwoldsche Aa are set respectively 36 m³/s, 11 m³/s and 4 m³/s.

3.5 Setting of model parameters

Part of the schematization process is the setting of various numerical and physical parameters. During the calibration process, some parameters, e.g. the bottom roughness and the time step were varied to check their impact on the calculated water levels. The following settings were assumed to perform best and are therefore applied for the Ems-Dollard Model:

- Water density: a uniform value of 1023 kg/m³, corresponding with a water temperature of 10°C and a salinity of 30 $^{0}/_{00}$

- Acceleration of gravity: 9.813 m/s²

- Corriolis parameter: corresponds to the latitude of 53.5° N

- Horizontal eddy viscosity: a uniform value of 2 m²/s

- Bottom roughness: according to the Manning formulation depth varying values are used ranging from 0.018 to 0.026 in the outer estuary and 0.012 to 0.019 in the Dollard embayment; along the Lower Ems depth varying Manning values are applied for particular blocks ranging from 0.011 to 0.015.

- Numerical time step: 30 seconds

4 Model calibration

4.1 Methodology

The aim of the model calibration is to adjust the model setting in such a way, that the simulated magnitudes at specific locations in the model domain match as best as possible with observations at corresponding locations in nature. To achieve this objective, a number of physical and numerical model parameters have to be modified and tuned. The variation of the parameters has to be kept within realistic ranges and assumptions.

During the calibration process more than one hundred simulations have been carried out with changes in bed roughness, resolution of the computational grid, model geometry, model bathymetry, discharges and a number of other numerical and physical parameters and settings. The computational effort is high considering the computation time of about 8 hours for each calibration run.

Model adjustments and parameter settings that result in a significant improvement of the model quality are described briefly:

Resolution of the computational grid

The resolution of the computational grid and thus the distance of the depth points interpolated on the grid points are very important for the model performance. A computational grid with coarse grid cells is inappropriate to resolve steep depth gradients in the bathymetry as e.g. in small tidal channels with steep slopes at their banks. On the other hand, exceeding the model resolution will result in computational times being uneconomical.

Initial model runs made evident, that the grid resolution in the Lower Ems was too low to resolve the cross profiles and hence the exchange of the total tidal volume appropriately. Due to the initial model setting, the water level in the Lower Ems did not sufficiently drop during ebb tide. Water levels did not decrease accordingly to observations, even not if a very smooth bottom roughness was introduced. Significant mismatches were observed around low water levels. Increasing the grid resolution in the Lower Ems solved these problems. An intensive model calibration was necessary for getting a balance between sufficient grid resolution and computational effort.

Model geometry

For a specific analysis of the hydrodynamics in the Lower Ems, the original computational grid that comprises the whole estuary from the seaward boundary to the tidal weir at Herbrum was split into two parts. Attention was paid on the landward part that covers the Lower Ems from Pogum to Herbrum. At the additional open model boundary at Pogum, the tidal forcing is imposed by measured water level time series. By splitting the model into two parts, the grid resolution in the Lower Ems could be increased by a factor of two in the cross flow direction without exceeding a reasonable size of the computational grid. The reduced model was used for a sensitivity analysis of the model parameters and a verification of the results for the overall model.
Using this set-up, the tributary stream Leda was schematized by a rectangular channel up to its tidal limit. The appropriate size and volume of this substitute-system could be evaluated more easily using the reduced model size. The conclusions drawn from the reduced but more detailed model of the Lower Ems were incorporated in the overall model.

Bed roughness adjustments

The bottom roughness is implemented in the model by use of the Manning formulation. Initial model settings were characterized by constant bottom roughness values in the entire model area, leading to rather poor results, particularily in the Lower Ems. In a second step the model area was subdivided into adjoining sections with respectively different bottom roughness values, but with a constant bottom roughness within each section. Sections more seaward were characterized by high Manning values, i.e. increased bottom roughness, and sections along the inner estuary by low Manning values, i.e. accounting for low bottom roughness. An extensive calibration was needed to figure out the most promising Manning value for each section.

To further improve the model performance, the bottom roughness values were related to the bottom depth. As a result, the bottom roughness values within each section vary spatially with the bottom depth. Deep water areas are exposed to less bottom roughness than shallow water areas.

4.2 Results

A time period for the model calibration of 60 hours was selected from 22nd of June 2005 12 p.m. to 25th of June 2005 12 a.m.. The period is characterized by low wind forces up to Beaufort 4 (5.5 to 7.9 m/s) in order to avoid impacts on water levels and varying wind directions (Fig. 9).

Only results of the simulation with the most promising settings are presented in order to limit the total number of figures. Computed and observed water levels are compared at seven tidal gauge locations along the estuary (Fig. 10a to 10g).

Average discrepancies between modeled and observed amplitudes are in the order of 5 to 10 centimeters with maximal deviations of up to 25 centimeters. Average phase lags between the modeled and observed water level peaks are less than 30 minutes.

Relative discrepancies between the computed and the measured tidal signal at Borkum Fischerbalje are found in a similar extent at other gauge locations along the inner estuary. This leads to the assumption, that most of the disagreement between the modeled and the observed tides can be attributed to offshore boundary conditions. Along the inner estuary, the propagation of the tidal wave is reproduced satisfactorily.

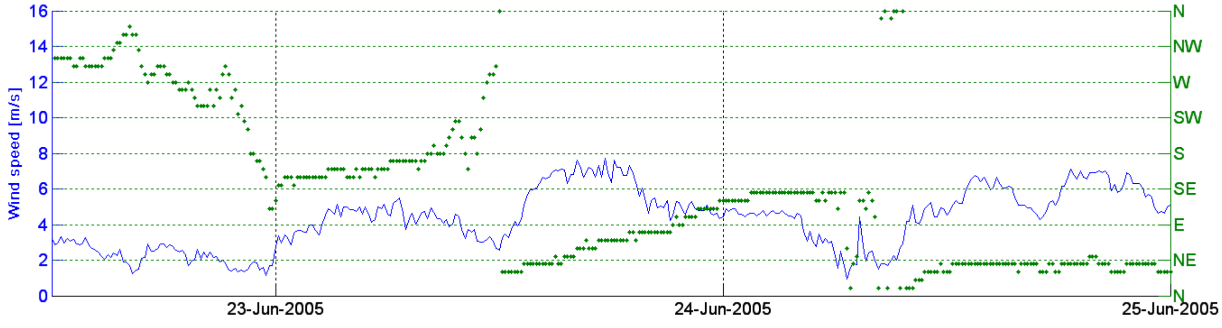

Fig. 9: Wind speed (m/s) and direction at Norderney during the time period used for calibration

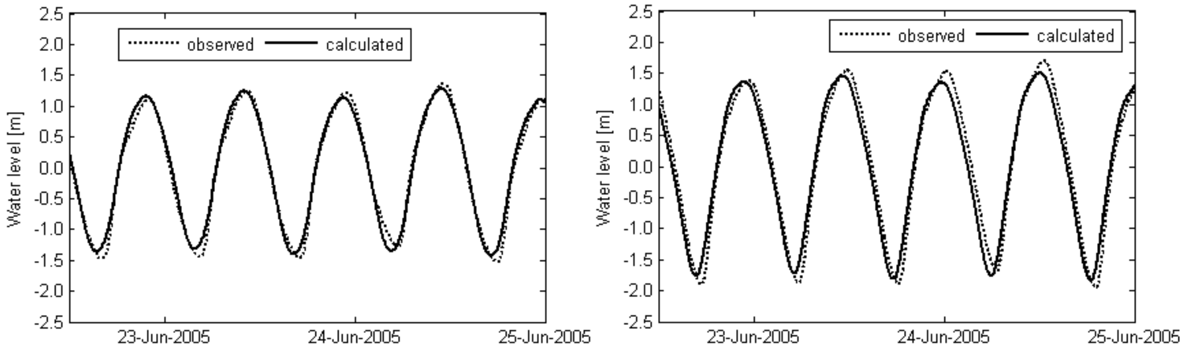

Fig. 10a: Borkum Fischerbalje

Fig. 10b: Knock

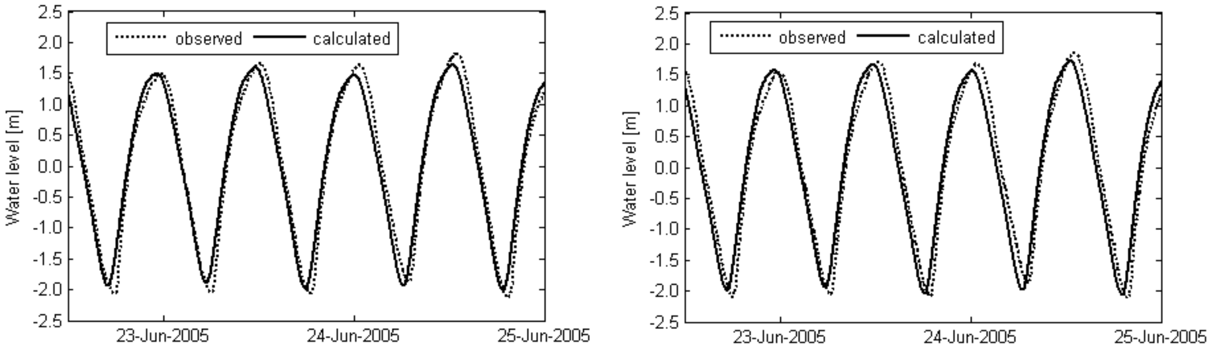

Fig. 10c: Emden Seeschleuse

Fig. 10d: Pogum

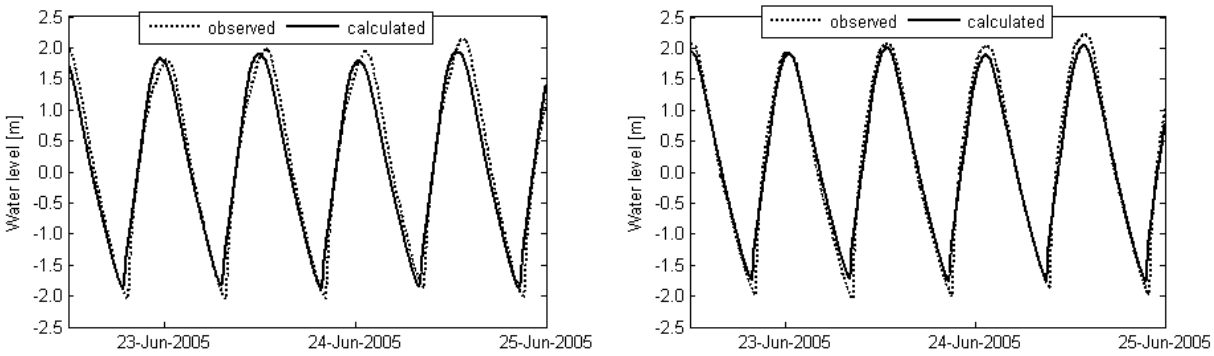

Fig. 10e: Leerort

Fig. 10f: Papenburg

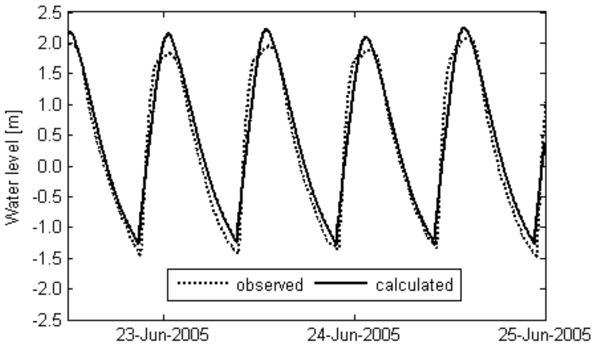

Fig. 10g: Herbrum (tidal barrier)

Fig. 10: Comparison of observed and computed water levels at tidal gauge locations along the Ems-Dollard estuary

5 Model validation

A model validation is usually carried out for a time period different than the period used for the calibration process. A full neap-spring-neap tidal cycle has been modeled covering the period from 29th of June to 14th of July 2005 for the water level gauges Borkum-Fischerbalje, Emden, Leerort, Papenburg and Herbrum (Fig. 12 to 14).

The discrepancies between modeled and observed water levels are to some extent higher than for the calibration process. This can be explained by the fact, that the meteorological conditions included wind forces of Beaufort 4 to 6 of varying directions (Fig. 11). The elevated wind speeds lead to water level set–ups and set–downs in the observed tide, whereas the modeled tide represents only the astronomical tide.

The comparison of computed and observed water levels shows the ability of the model to reproduce the prevailing astronomical and physical conditions satisfactorily for astronomical tides.

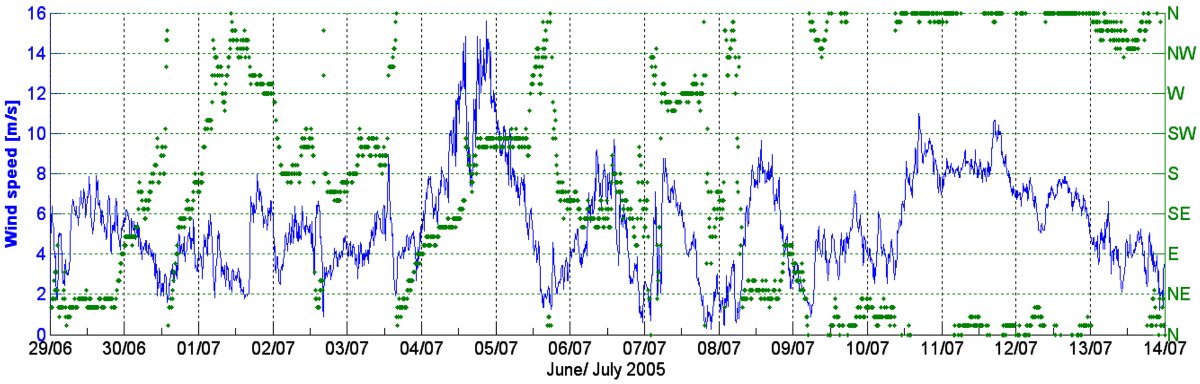

Fig. 11: Wind speed (m/s) and direction at Norderney during the period used for calibration

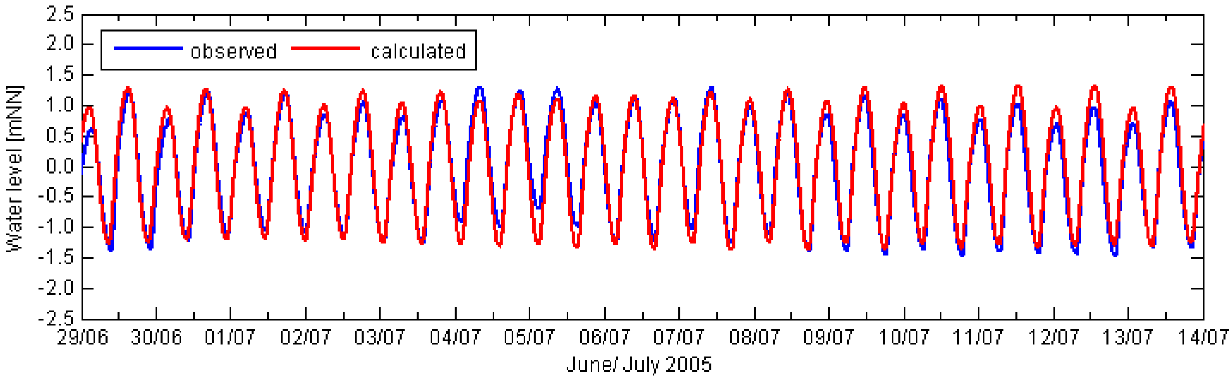

Fig. 12: Comparison of observed and computed water levels at gauge location Borkum Fischerbalje

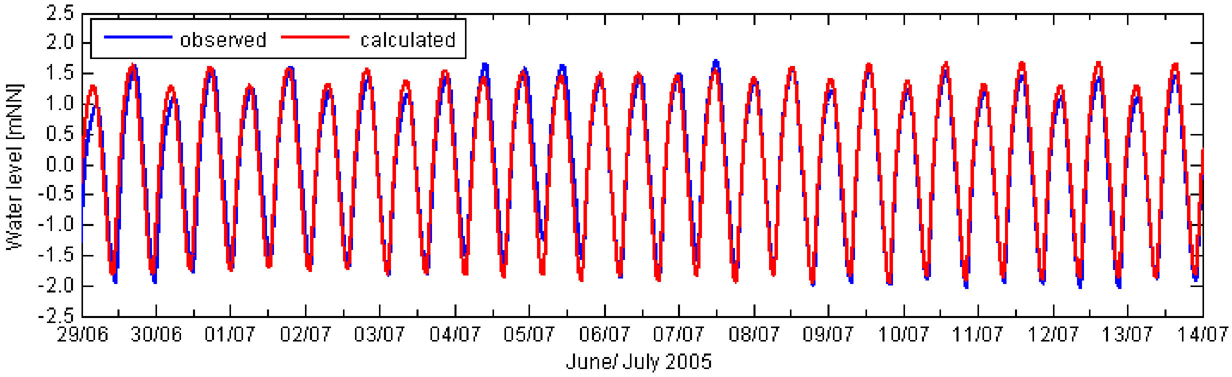

Fig. 13: Comparison of observed and computed water levels at gauge location Emden Neue Seeschleuse

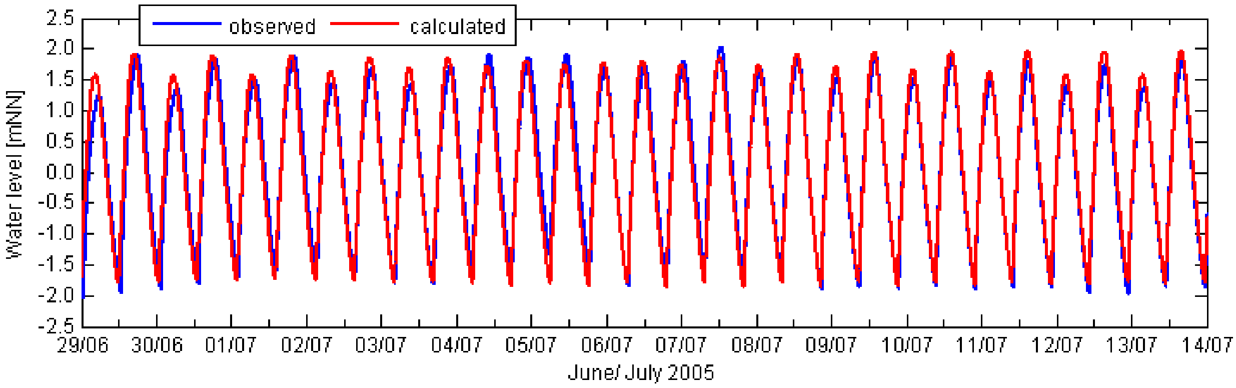

Fig. 14: Comparison of observed and computed water levels at gauge location Leerort

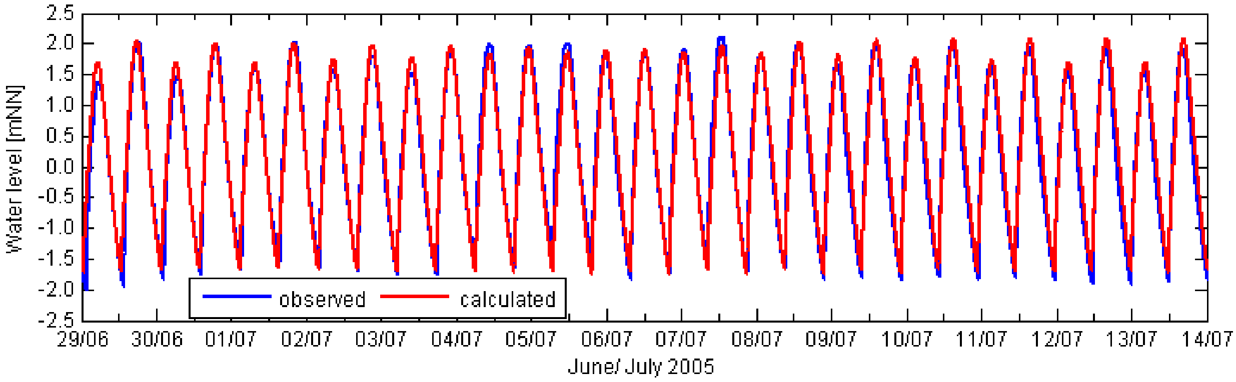

Fig. 15: Comparison of observed and computed water levels at gauge location Papenburg

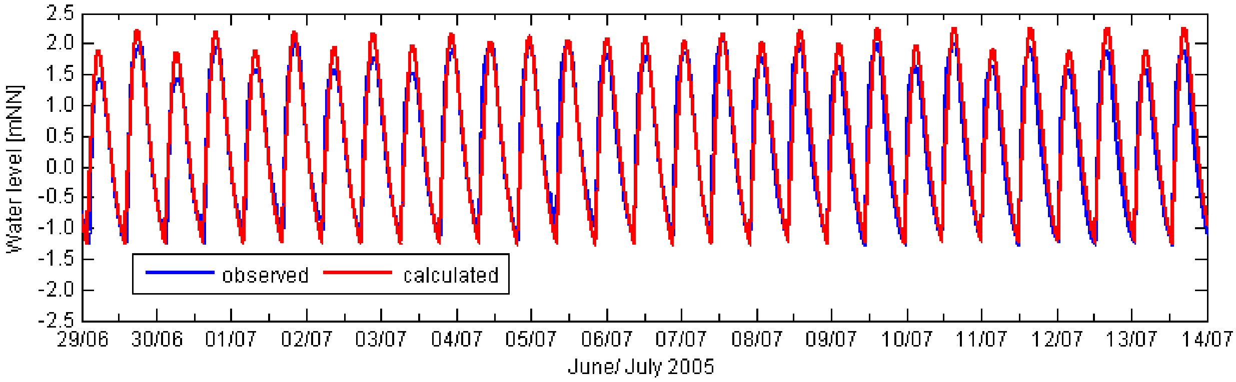

Fig. 16: Comparison of observed and computed water levels at gauge location Herbrum Hafendamm

6 Summary

A hydrodynamic model of the Ems-Dollard estuary was established by applying the vertically averaged version of the modeling system Delft3D (DELFT HYDRAULICS, 2006).

A high-resolution computational grid was generated covering the Ems-Dollard estuary including the river Ems downstream of the tidal barrier at Herbrum, the Dollard Bay and the outer Ems. A bathymetry with recent data of topographic surveys based on echo-sounding and airborne laser-scanning was used for the model bathymetry.

The Ems-Dollard Model is driven by water level and current velocity time series obtained by a nesting procedure with the existing overall German Bight Model. To reveal the quality of the generated boundary conditions, tidal signals based on long-term observations and on calculations of the nested model are analyzed by means of a harmonic analysis.

For the primary constituents, the comparison of decomposed partial tides shows maximal discrepancies in the order of 4 % (M2) and 9% (N2) between the computed and observed values. Higher percental discrepancies occur for the secondary constituents, but the ascribed weight to the composed tide is less significant as the amplitudes are smaller.
The divergence is regarded as small compared to those incorporated due to meteorological effects. The offshore boundary conditions from nesting with the German Bight Model are suitable for forcing the estuarine model.

Hydrodynamic simulations have been performed and computed water levels were compared to observations of existing water level gauges along the estuary. An extensive model calibration was needed to adjust model configurations and uncertain parameters to optimize the matching of the model with observations.

For the period of a neap-spring-neap cycle, the model settings determined in the calibration process are verified satisfactorily with respect to water level measurements. The configuration of the hydrodynamic model of the Ems-Dollard estuary will be applied as a basis for further hydrodynamical investigations of historical topographic states and for morphodynamical computations.

7 Literature

JONGE, V.N. de, (2000): Importance of spatial and temporal scales in applying biological and physical process knowledge in coastal management, an example for the Ems estuary. Continental Shelf Research, 20, pp. 1655-1686.

PANSCH, E. (1989): New Harmonic Tidal Constants for some German North Sea Stations, Deutsche Hydrographische Zeischrift, 42. Jg. 1989, H. 1, pp. 27-39.

SCHUBERT, K. (1970): Ems und Jade, Die Küste, Heft 19.

VERBOOM, G.K.; RONDE, J.G. de; DIJK; R.P. van (1992): A Fine Grid Tidal Flow and Storm Surge Model of the North Sea. Continent. Shelf Res., Vol. 12.

WL|Delft Hydraulics (1997): Set-up and Calibration of Tidal Flow Models Deutsche Bucht und Dithmarschen Bucht. Rapp. H 1821 (not published)

WL|Delft Hydraulics (2006): Delft3D-Flow User Manual

WL|Delft Hydraulics: www.delft-hydraulics.com